

Time Of Death ?

Suppose that you come into your Math teacher's office to ask him some questions shortly before 9 :00 a.m. on Friday. You find him lying on the floor of his office on chalk dust, dead.

You quickly call the police and their investigators collect the following data :

- the body temperature at 9 :00 a.m. 26.6 C ;
- the body temperature at 10h00 a.m. 25.5 C ;
- the room temperature 21.1 C.

You realize that the police believe you to be a prime suspect, so you need an alibi. You know that you were studying until midnight, but you aren't sure if that is enough information. You need to know the time of death! You know that the difference between body temperature and room temperature changes at a rate proportional to that difference. This physical law is called "**Newton's law of cooling**".

Let $\theta(t)$ be the body temperature (measured in °C) at the time t (measured in hours). It gives the following differential equation ¹ :

$$\frac{d\theta}{dt} = a(\theta - 21.1)$$

We can rewrite this differential equation like this :

$$(E) : \theta' - a\theta = -21.1a$$

1. The differential equation (E) is a first order linear equation, so the solutions look like $\theta(t) = ke^{-at} + 21.1$ where k is a constant.

Use the body temperature at 9 :00 and 10 :00 to find the constants a and k .

2. Let $f(t) = 41e^{-0.223t} + 21.1$ for $3.5 \leq t \leq 12$.
 - (a) Find f' the derivative of f , and show that f is decreasing.
 - (b) Sketch the curve \mathcal{C}_f .
 - (c) Use the curve to solve approximately the equation $f(x) = 37.2$.
And finally find the time of death of your beloved Math teacher.
3. How good is your alibi ?

Glossary

- Chalk : craie.
- Rate of change : limite du taux de variation c'est-à-dire nombre dérivé.
- Increasing/decreasing : croissant/décroissant.
- Sketch : tracer.

¹A differential equation is an equation which contains an unknown function and one of its unknown derivatives.

